Context-free Categorical Grammars

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Plan

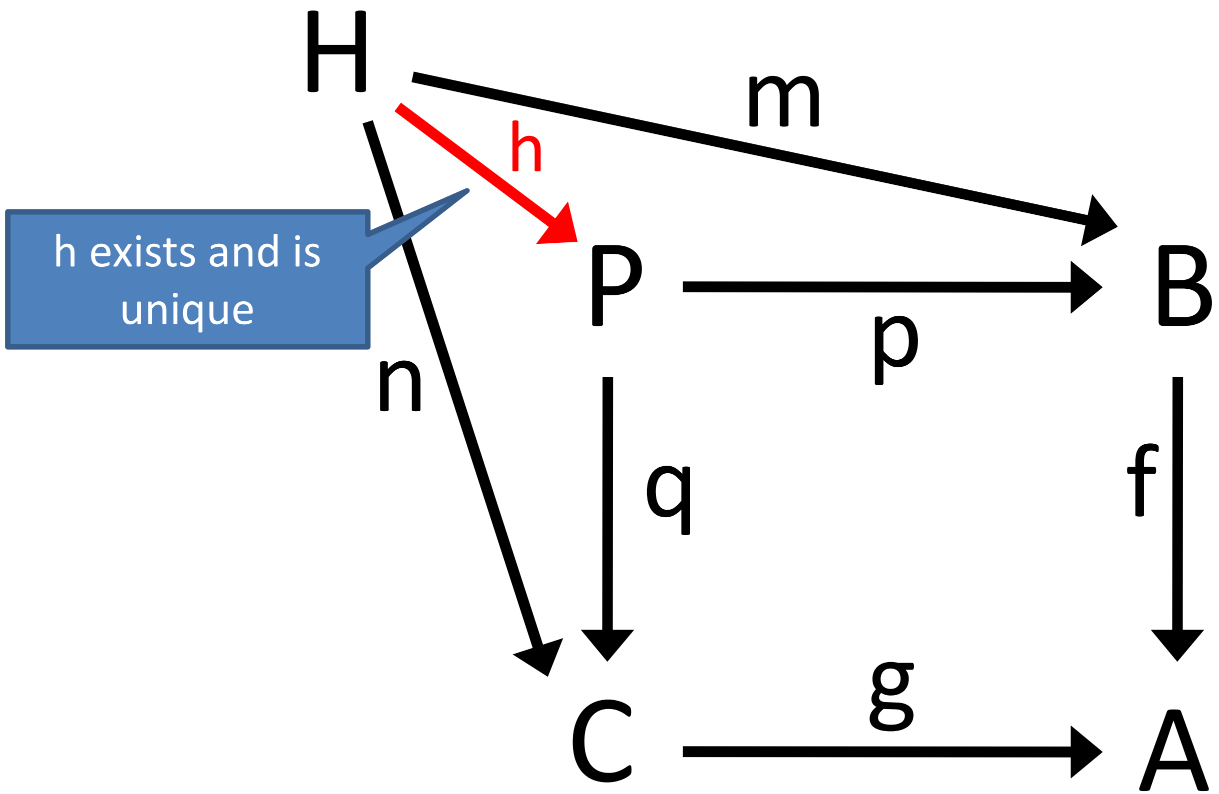
1. Categories, Pullback and Pushout
2. Categorical Grammars
3. Words Grammars
4. Context-free ness

CATEGORIES,
PULLBACK & PUSHOUT

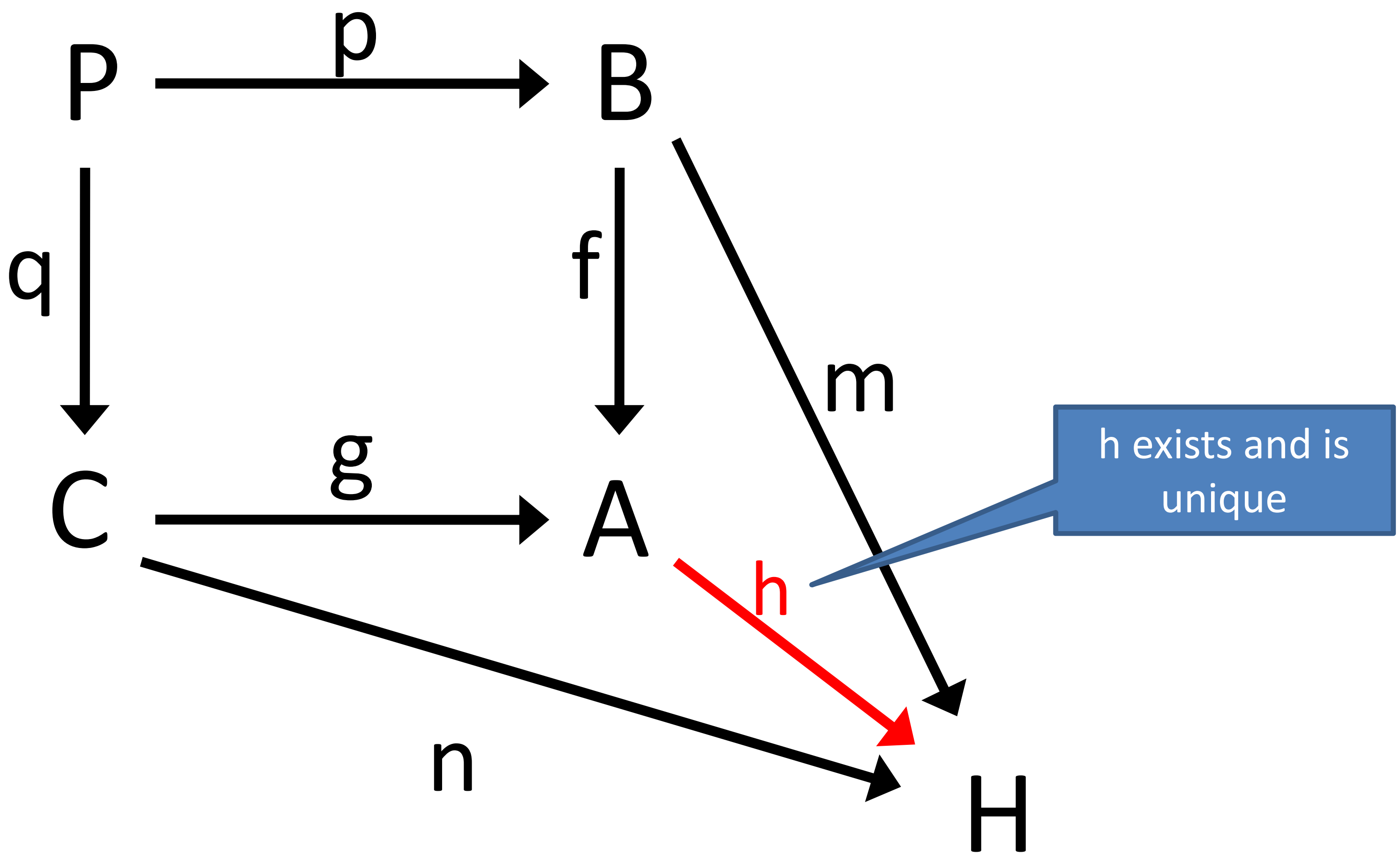
Category

- Objects
- Arrows (morphims)

Pullback

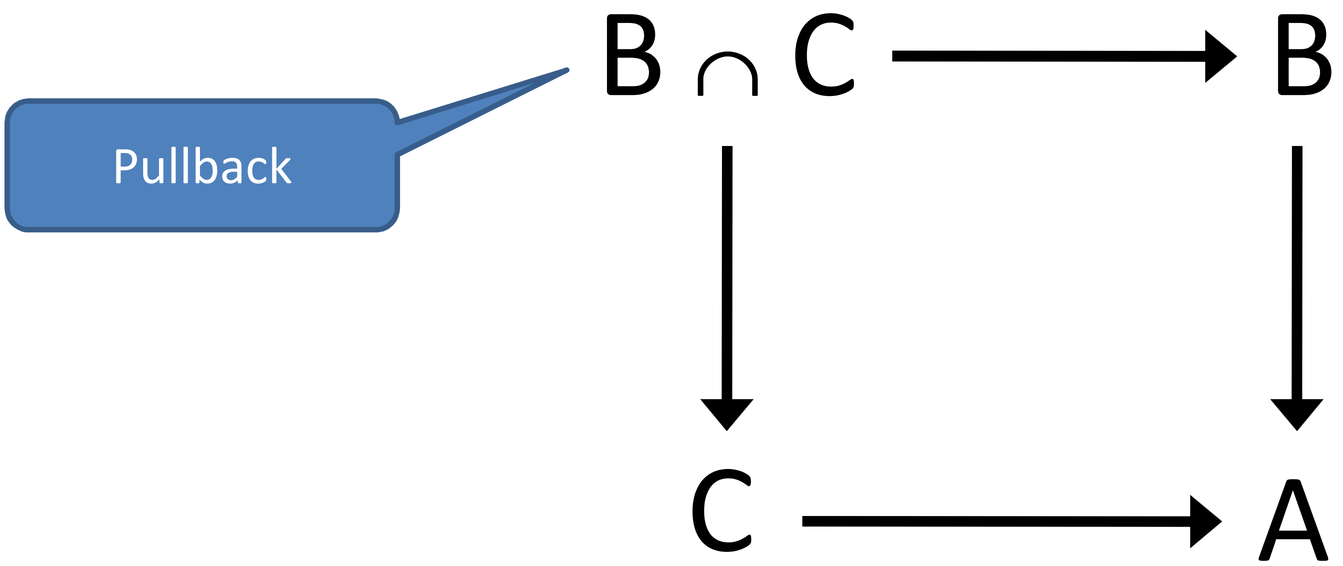


Pushout



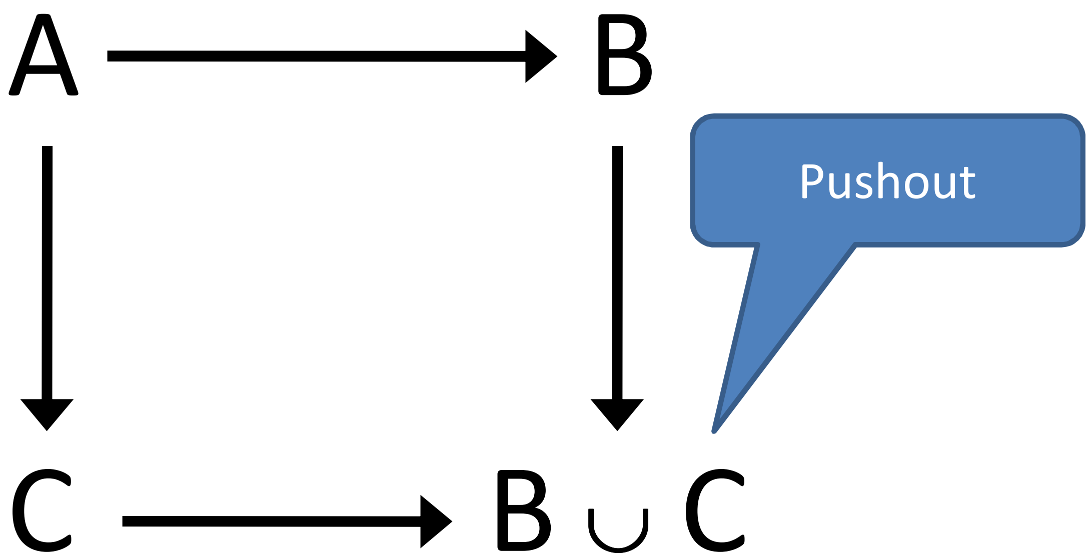
Example

- Objects Sets (A, B, C, …)
- Arrows $A \rightarrow B$ iff $A \subseteq B$



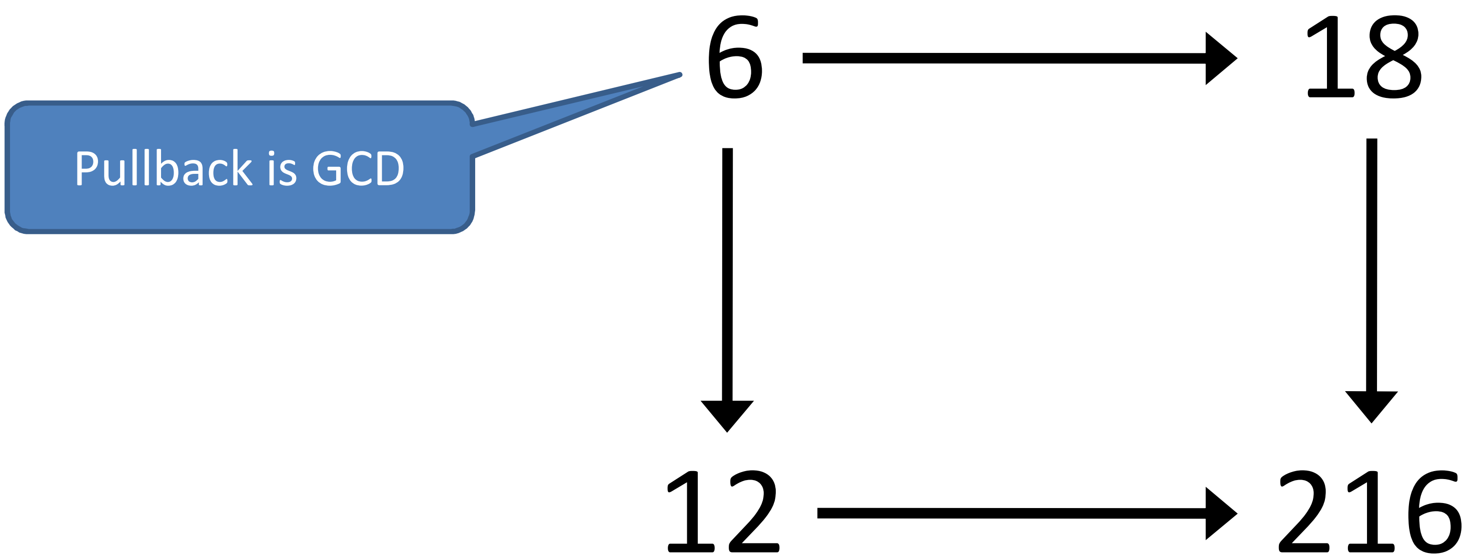
Example 1 : Sets and inclusion

- Objects — Sets (A, B, C, …)
- Arrows — $A \rightarrow B$ iff $A \subseteq B$



Example 2 : Natural numbers and division

- Objects : positive integers
- Arrows : $n \to m$ iff $n|m$



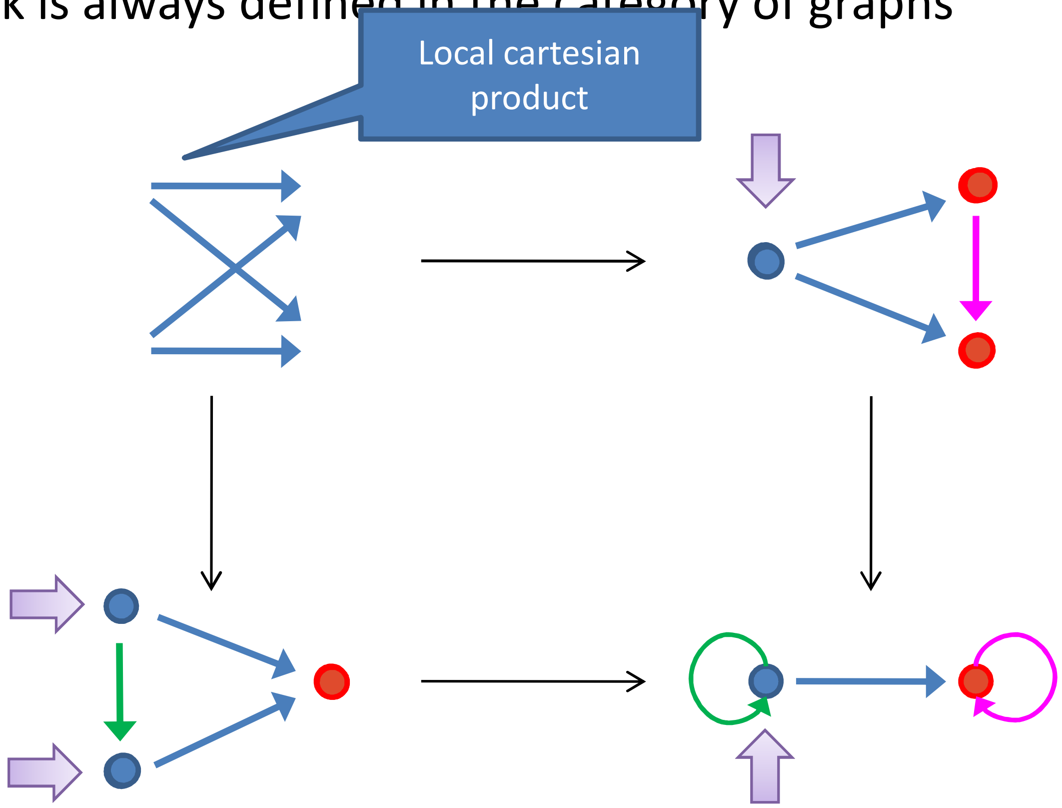
Example 3 : The category of graphs

- Graphs: $\langle V, E, s, t \rangle$ with
  $s : E \to V$ defines sources of the edges
  $t : E \to V$ defines targets of the edges
  } objects

- Graph morphisms $\langle f_V, f_E \rangle : G_1 \to G_2$ with
  $f_V : V_{G_1} \longrightarrow V_{G_2}$
  $f_E : E_{G_1} \longrightarrow E_{G_2}$
  } arrows

Pullback : a mechanism for graph rewriting [Bauderon 95]

The pullback is always defined in the category of graphs



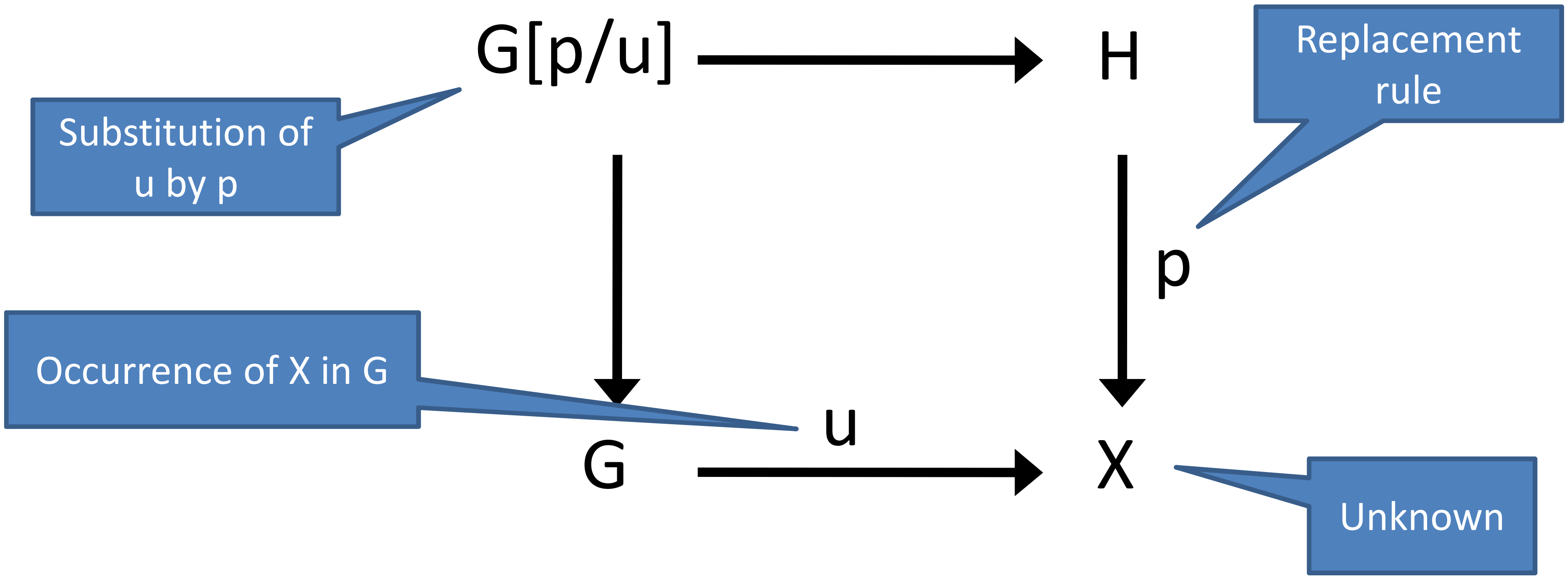
CATEGORICAL GRAMMARS

Unknowns & Substitutions

Let **C** be a category.

Let $G \xrightarrow{u} X \xleftarrow{p} H$ be a span in **C**.

G[p/u] denotes the associated pullback.



Sources

Let $N \subset C$ be a subset of C-objects : the *non-terminals.*

A N –source is a triple

Formally: U_G and P_G are collection of arrows of form $G \rightarrow X$ where $X \in N$

$$\overline{G} = (G, U_G, P_G)$$

An *object* of C

A collection of *unknown occurrences* in G

A collection of *replacement rules* with G as right-member

Substitution

Let $\overline{G} = (G, U_G, P_G)$ and $\overline{H} = (H, U_H, P_H)$ be 2 sources

Let $u : G \rightarrow X \in U_G$ be an unknown of G

Let $p : H \rightarrow X \in P_G$ be a replacement rule of H

The *substitution* of u by p in G is defined to be

$$\overline{G}[p/u] = (G[p/u], (U_G \setminus \{u\})[p/u] \cup U_H[u/p], P_G[p/u])$$

Substitution in the object

Transformations of unknown occurences

Addition of the unknowns of H

Transformations of replacement rule based on G

Duality

One obtains a dual definition by replacing

- Span by co-span
- Pullback by pushout

Categorical Grammars

A **rewriting rule** is a pair X $\Rightarrow$ G where X is a non-terminal and G is a source.

A **rewriting system** is a set of rewriting rule.

A **grammar** is a rewriting system provided with a distinguished non-terminal called the axiom.

Grammar based on pullback are called **projective grammars**.

Grammar based on pushout are called **inductive grammars**.

WORDS CATEGORICAL GRAMMARS

Words as a category

Words:

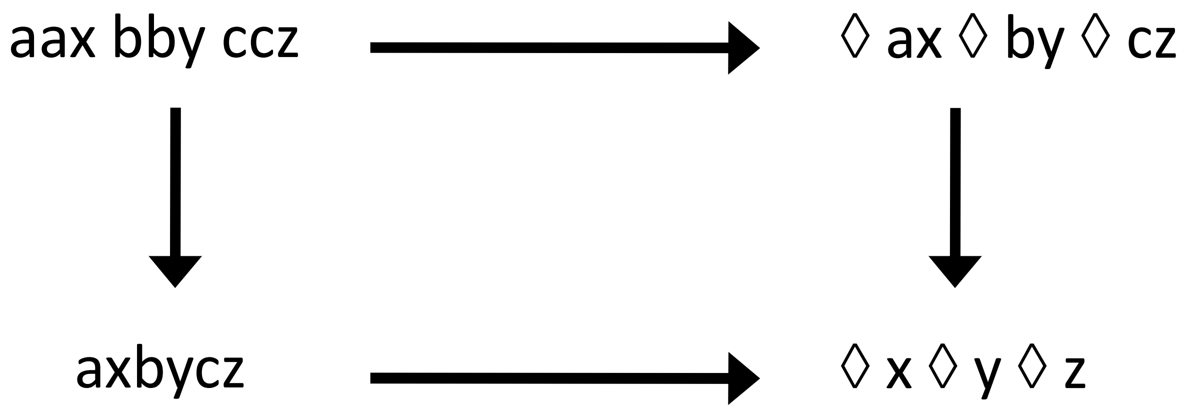
Totally ordered finite sets labeled over an alphabet.

Result:

- Inductive languages = Context-free languages
- Context-free languages $\subsetneq$ Projective languages.

$a^n b^n c^n$ is projective

$\Diamond ax \Diamond by \Diamond cz \rightarrow \Diamond x \Diamond y \Diamond z$



THE EXAMPLE OF GRIDS

CATEGORICAL GRAMMARS ARE CONTEXT-FREE

Some open questions

- Decision questions about the logic of projective graph grammars.
- HR graph grammars correspond to tree-width, to what projective ones do correspond ?