Single-Path Restarting Tree Automata

Friedrich Otto and **Heiko Stamer**

Universität Kassel
Fachbereich Elektrotechnik/Informatik
Wilhelmshöher Allee 71-73, 34109 Kassel, Germany

otto@theory.informatik.uni-kassel.de

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Outline

Analyis by Reduction

Linguistic task: Analyze sentences of a natural language

- verify syntactic correctness
- determine dependencies
- disambiguate between morphological ambiguities

A linguistic technique: Analysis by reduction

Application: natural languages with a free word order
(Czech, Polish, Russian, Turkish, ...)

Restarting Automata on Strings

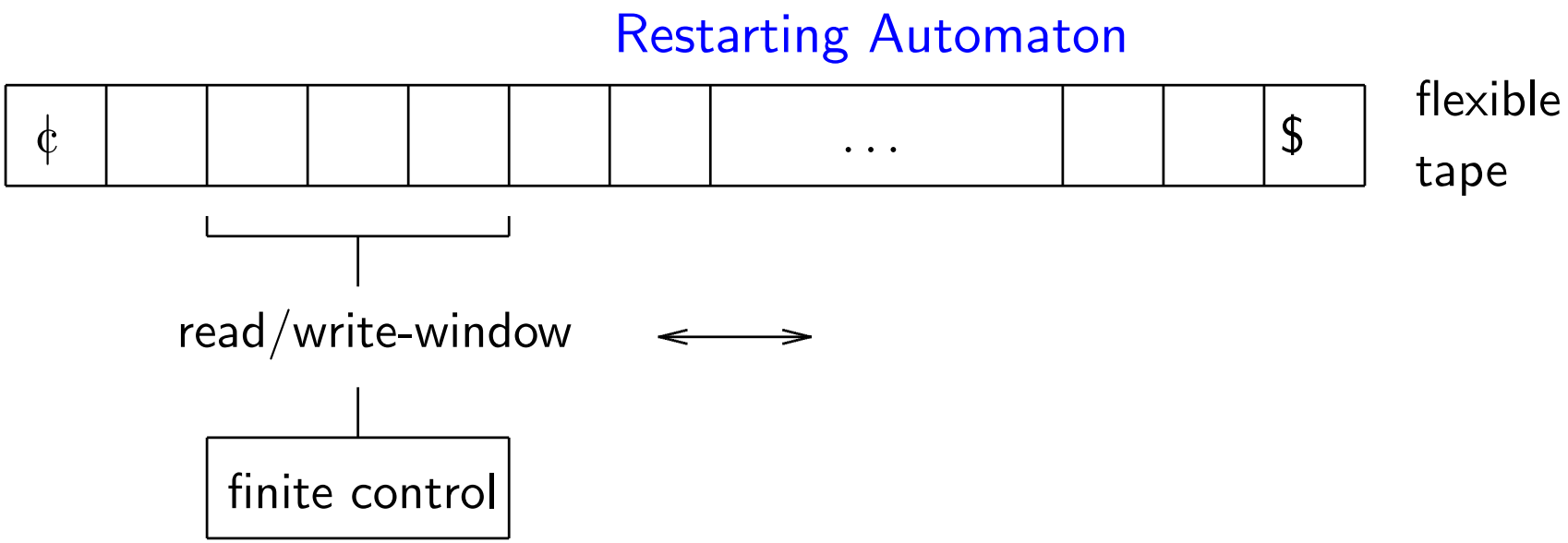
Operations used in the Analysis by Reduction:

Move-right: MVR

Delete/Rewrite: replace a small part of the sentence

Restart: start anew on a simplified sentence

Jančar, Mráz, Plátek, Vogel (1995 – 1999):



Restarting Tree Automata

Roughly speaking, a *Restarting Tree Automaton* is an

iterated top-down tree automaton

equipped with a

height-bounded read/write-window

and the capability to

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Restarting Tree Automata

A computation consists of finitely many consecutive cycles of the form:

1. Process the input beginning at the root and fork down to the leaves using a regular control similar to a top-down finite tree automaton.
2. On the way to the leaves, perform *at most* one size-reducing rewrite step on each separate branch.
3. Depending on an additional regular control,
   - halt and reject the input, or
   - execute a restart, i.e., move the read/write-window to the top and start with the initial state (i.e., forget all information collected).

} cycle

Repeat this cycle until a "simple form" of the input is obtained, which can be recognized in step 1 only.

Definition (Restarting Tree Automaton)

An RRWWT-automaton is a six-tuple $\mathcal{A} = (\mathcal{F}, \mathcal{G}, \mathcal{Q}, q_0, k, \Delta)$,

- $\mathcal{F}$ is a ranked input alphabet, $\mathcal{G} \supseteq \mathcal{F}$ is a ranked working alphabet,
- $\mathcal{Q} = \mathcal{Q}_1 \cup \mathcal{Q}_2$ is a finite set of states such that $\mathcal{Q}_1 \cap \mathcal{Q}_2 = \emptyset$,
- $q_0 \in \mathcal{Q}_1$ is the *initial state* and simultaneously the *restart state*,
- $k \geq 1$ is the height of the read/write-window, and
- $\Delta = \Delta_1 \cup \Delta_2$ is a finite term rewriting system (TRS) on $\mathcal{G} \cup \mathcal{Q}$.

(Stateless) configuration: ground term from $\mathcal{T}(\mathcal{G} \cup \mathcal{Q})$ (resp. from $\mathcal{T}(\mathcal{G})$)

$\rightarrow_\Delta$ ($\rightarrow_{\Delta_1}$) denotes the *move relation* induced by the TRS Δ (resp. Δ_1)

$\hookrightarrow_\mathcal{A}$ *cycle relation*: for $u, v \in \mathcal{T}(\mathcal{G})$, $u \hookrightarrow_\mathcal{A} v$ iff $q_0(u)(\rightarrow_\Delta^+ \setminus \rightarrow_{\Delta_1}^+)v$.

$$L(\mathcal{A}) = \left\{ t_0 \in \mathcal{T}(\mathcal{F}) \;\middle|\; \begin{array}{l} \exists \ell \geq 0, \exists t_1, \ldots, t_\ell \in \mathcal{T}(\mathcal{G}) \text{ such that} \\ t_0 \hookrightarrow_\mathcal{A} t_1, \ldots, t_{\ell-1} \hookrightarrow_\mathcal{A} t_\ell, \text{ and } q_0(t_\ell) \rightarrow_{\Delta_1}^+ t_\ell \end{array} \right\}$$

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Form of the Rules from Δ_1 and Δ_2

Δ_1 contains only the following *linear* transition rules:

1. Top-down transitions: $q(t) \to t[q_1(x_1), \ldots, q_n(x_n)]$
$(n \geq 1,\ t \in \mathrm{Ctx}(\mathcal{G}, \mathcal{X}_n),\ 1 \leq \mathrm{Hgt}(t) \leq k,\ q, q_1, \ldots, q_n \in \mathcal{Q}_1)$
2. Final transitions: $q(t) \to t$ $(t \in \mathcal{T}(\mathcal{G}),\ 0 \leq \mathrm{Hgt}(t) \leq k,\ q \in \mathcal{Q}_1)$

Δ_2 contains only the following *linear* transition rules:

1. *Top-down transitions:* $q(t) \to t[q_1(x_1), \ldots, q_n(x_n)]$
$(n \geq 1,\ t \in \mathrm{Ctx}(\mathcal{G}, \mathcal{X}_n),\ 1 \leq \mathrm{Hgt}(t) \leq k,\ q, q_1, \ldots, q_n \in \mathcal{Q}_2)$
2. *Final transitions:* $q(t) \to t$ $(t \in \mathcal{T}(\mathcal{G}),\ 0 \leq \mathrm{Hgt}(t) \leq k,\ q \in \mathcal{Q}_2)$
3. *Size-reducing rewrite transitions:* $q(t) \to t'[q_1(x_1), \ldots, q_n(x_n)]$
$(n \geq 1,\ t, t' \in \mathrm{Ctx}(\mathcal{G}, \mathcal{X}_n),\ ||t|| > ||t'||,\ \mathrm{Hgt}(t) \leq k,$
$q \in \mathcal{Q}_1,\ q_1, \ldots, q_n \in \mathcal{Q}_2)$
4. *Size-reducing final rewrite transitions:* $q(t) \to t'$
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Results

Theorem (Stamer, Otto, SOFSEM 2007)

The non-context-free tree language

$$L_4 = \{ f(g^i(h^i(a)), g^i(h^i(a))) \mid i \geq 1 \}$$

is accepted by an RT*-automaton.*

Theorem (Stamer, Otto, CAI 2007)

Given a linear context-free tree grammar G, an RWWT*-automaton $\mathcal{A}$ can be constructed such that $L(G) = L(\mathcal{A})$.*

Single-Path Restriction

I. Single-Path Restriction

Single-Path Top-Down Tree Automaton

Definition (Single-Path Top-Down Tree Automaton)

An spNF↓T-automaton is a five-tuple $\mathcal{A} = (\mathcal{F}, \mathcal{Q}, q_0, k, \Delta)$, where

- $\mathcal{F}$ is a ranked alphabet, $\mathcal{Q}$ is a finite set of states, $q_0 \in \mathcal{Q}$ is the *initial state*,
- $k \geq 1$ is the height of the *look-ahead* window, and
- Δ is a finite term rewriting system (TRS) on $\mathcal{F} \cup \mathcal{Q}$ that contains only
  1. k-height bounded linear *top-down transitions* of the form

$$q(t) \to t[x_1, \ldots, x_{i-1}, q'(x_i), x_{i+1}, \ldots, x_n], \text{ where}$$

  $n \geq 1$, $t \in \mathrm{Ctx}(\mathcal{F}, \mathcal{X}_n)$, $\mathrm{Hgt}(t) \leq k$, $q, q' \in \mathcal{Q}$, $i \in \{1, \ldots, n\}$, and
  2. k-height bounded *final transitions* of the form $q(t) \to t$, where $t \in \mathcal{T}(\mathcal{F})$, $\mathrm{Hgt}(t) \leq k$, and $q \in \mathcal{Q}$.

The recognized tree language is $L(\mathcal{A}) = \{t \in \mathcal{T}(\mathcal{F}) \mid q_0(t) \to_\Delta^* t\}$.

Single-Path Top-Down Tree Automaton

$\mathscr{L}(\mathrm{FINT}) \subsetneq \mathscr{L}(\mathrm{spNF}{\downarrow}\mathrm{T}) \subsetneq \mathscr{L}(\mathrm{RTG})$ holds,

but $\mathscr{L}(\mathrm{spNF}{\downarrow}\mathrm{T})$ is incomparable to $\mathscr{L}(\mathrm{DF}{\downarrow}\mathrm{T})$ under set inclusion,

as the language

$$L_d := \{\, f(g^i(a), g^j(b)) \mid i, j \geq 0 \,\} \in \mathscr{L}(\mathrm{DF}{\downarrow}\mathrm{T})$$

is not recognized by any $\mathrm{spNF}{\downarrow}\mathrm{T}$.

Single-Path Restarting Tree Automaton

Definition (Single-Path Restarting Tree Automaton)

An spRRWWT-automaton is a six-tuple $\mathcal{A} = (\mathcal{F}, \mathcal{G}, \mathcal{Q}, q_0, k, \Delta)$,

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where $\mathcal{A}_{S_\mathcal{G}} = (\mathcal{G}, \mathcal{Q}_1, q_0, k, \Delta_1)$ is an spNF$\downarrow$T-automaton on $\mathcal{G}$.

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   $(n \geq 1,\ t \in \mathrm{Ctx}(\mathcal{G}, \mathcal{X}_n),\ 1 \leq \mathrm{Hgt}(t) \leq k,\ q, q' \in \mathcal{Q}_1,\ i \in \{1, \ldots, n\})$
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4. Size-reducing final rewrite transitions: $q(t) \to t'$
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Basic Variants of (sp)RRWWT-Automata

Basic variants of (sp)RRWWT-automata, derived from the corresponding restarting automata on words:

RR-prefix: General case, no restriction.

R-prefix: Every *rewrite transition* contains no successor state(s) in the corresponding RHS, i.e. $q(t) \to t'[x_1, \ldots, x_n]$.
That means, after a rewrite step no additional regular control can be performed in the affected branches.

WW-suffix: General case, no restriction.

W-suffix: The working alphabet $\mathcal{G}$ coincides with the input alphabet $\mathcal{F}$, i.e., there are no auxiliary symbols available.

λ-suffix: The term t' of the RHS of every *rewrite transition* is homeomorphically embedded in the LHS.

Expressive Power

Example ($L_4 = \{f(g^i(h^i(a)), g^i(h^i(a))) \mid i \geq 1\} \notin \mathscr{L}(\mathrm{CFTG})$)

RT-automaton: $\mathcal{F} = \mathcal{G} = \{f(\cdot,\cdot), g(\cdot), h(\cdot), a\}$ (no auxiliary symbols), $k = 3$, $Q = Q_1 \cup Q_2$, where $Q_1 = \{ q_0 , q_1 \}$ and $Q_2 = \emptyset$,

$$\Delta_1 = \left\{ \begin{array}{ccc} q_0 & & \\ f & & f \\ g\ \ g & \to & g\ \ g \\ h\ \ h & & h\ \ h \\ a\ \ a & & a\ \ a \end{array}, \begin{array}{ccc} q_0 & & \\ f & & f \\ g\ \ g & \to & q_1\ \ q_1 \\ x_1\ \ x_2 & & x_1\ \ x_2 \end{array}, \begin{array}{ccc} q_1 & & \\ g & & g \\ x_1 & \to & x_1 \end{array} \right\},$$

$$\Delta_2 = \left\{ \begin{array}{ccc} q_1 & & \\ g & & \\ h & \to & h \\ h & & x_1 \\ x_1 & & \end{array} \right\}.$$

Expressive Power

Example ($L_1 = \{f(g^i(a), g^i(a)) \mid i \geq 0\} \in \mathscr{L}(\mathrm{CFTG}) \setminus \mathscr{L}(\mathrm{RTG})$)

spRT-automaton: $\mathcal{F} = \mathcal{G} = \{f(\cdot,\cdot), g(\cdot), a\}$ (no auxiliary symbols), $k = 2$, $Q = Q_1 \cup Q_2$, where $Q_1 = \{ q_0 \}$ and $Q_2 = \emptyset$,

$$\Delta_1 = \left\{ \begin{array}{ccc} q_0 & & \\ f & \to & f \\ a\ \ a & & a\ \ a \end{array} \right\},$$

$$\Delta_2 = \left\{ \begin{array}{ccc} q_0 & & \\ f & & f \\ g\ \ g & \to & x_1\ \ x_2 \\ x_1\ \ x_2 & & \end{array} \right\}.$$

Proposition

Let $X \in \{\mathrm{R}, \mathrm{RR}, \mathrm{RW}, \mathrm{RRW}, \mathrm{RWW}, \mathrm{RRWW}\}$ be a type of restarting automaton. Then for each sp$X$T-automaton $\mathcal{A}$ there exists an XT-automaton $\mathcal{A}'$ such that $L(\mathcal{A}) = L(\mathcal{A}')$.

Expressive Power

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$$\Delta_2 = \left\{ \begin{array}{ccc} q_0 & & \\ f & & f \\ g\ \ g & \to & x_1\ \ x_2 \\ x_1\ \ x_2 & & \end{array} \right\}.$$

Proposition

Let $X \in \{\mathrm{R}, \mathrm{RR}, \mathrm{RW}, \mathrm{RRW}, \mathrm{RWW}, \mathrm{RRWW}\}$ be a type of restarting automaton. Then for each sp$X$T*-automaton $\mathcal{A}$ there exists an XT-automaton $\mathcal{A}'$ such that $L(\mathcal{A}) = L(\mathcal{A}')$.*

Regular Tree Languages

Theorem

$\mathscr{L}(\mathrm{RTG}) \subsetneq \mathscr{L}(\mathrm{spRT})$.

Proof (idea):

- Simulate a complete DF↑T-automaton $\mathcal{B} = (\mathcal{F}, Q^{(\mathcal{B})}, Q_f^{(\mathcal{B})}, \Delta^{(\mathcal{B})})$ by an spRT-automaton $\mathcal{A} := (\mathcal{F}, \mathcal{F}, Q, q_0, k, \Delta)$ where $k := |Q^{(\mathcal{B})}| + 1$
- $\mathcal{A}$ accepts all $t \in L(\mathcal{B})$ satisfying $\mathrm{Hgt}(t) \leq k$ by final transitions
- Construct final rewrite transitions of the form $q(t) \to t_1[t_3]$, for all $t \in \mathcal{T}(\mathcal{F})$, where $\mathrm{Hgt}(t) = k$ and $t = t_1[t_2[t_3]]$, such that

$$t_3 \to^*_{\Delta^{(\mathcal{B})}} p \quad \text{and} \quad t_2[t_3] \to^*_{\Delta^{(\mathcal{B})}} p,$$

for some $p \in Q^{(\mathcal{B})}$ (pumping argument, note that $\mathcal{B}$ is deterministic)

Linear Context-Free Tree Languages

Theorem

Given a linear context-free tree grammar G, an spRWWT*-automaton $\mathcal{A}$ can be constructed such that $L(G) = L(\mathcal{A})$.*

Proof (idea): (Stamer and Otto, CAI 2007)

- Transform G into a normal form with only *growing* productions
- Simulate all derivations of G nondeterministically and in reverse order
- This simulation can be done with an spRWWT-automaton as well

Corollary

$\mathscr{L}(\text{lin-CFTG}) \subsetneq \mathscr{L}(\text{spRWWT})$.

Ground-Rewrite Restriction

II. Ground-Rewrite Restriction

Ground-Rewrite Restarting Tree Automaton

Definition (Ground-Rewrite Restarting Tree Automaton)

A gr-RWWT-automaton $\mathcal{A} = (\mathcal{F}, \mathcal{G}, \mathcal{Q}, q_0, k, \Delta)$ is essentially an RWWT-automaton for which Δ_2 only contains *final* rewrite transitions.

Example ($L_1 = \{f(g^i(a), g^i(a)) \mid i \geq 0\} \in \mathscr{L}(\mathrm{CFTG}) \setminus \mathscr{L}(\mathrm{RTG})$)

gr-RT-automaton: $\mathcal{F} = \mathcal{G} = \{f(\cdot,\cdot), g(\cdot), a\}$, $k = 1$, $\mathcal{Q} = \mathcal{Q}_1 \cup \mathcal{Q}_2$, where $\mathcal{Q}_1 = \{ q_0 , q_1 \}$ and $\mathcal{Q}_2 = \emptyset$,

$$\Delta_1 = \left\{ \begin{array}{c} q_0 \\ f \\ x_1 \;\; x_2 \end{array} \to \begin{array}{c} f \\ q_1 \;\; q_1 \\ x_1 \;\; x_2 \end{array}, \quad \begin{array}{c} q_0 \\ f \\ a \;\; a \end{array} \to \begin{array}{c} f \\ a \;\; a \end{array}, \quad \begin{array}{c} q_1 \\ g \\ x_1 \end{array} \to \begin{array}{c} g \\ q_1 \\ x_1 \end{array} \right\},$$

$$\Delta_2 = \left\{ \begin{array}{c} q_1 \\ g \\ a \end{array} \to a \right\}.$$

Single-Path Versus Ground Rewrite Restriction

Let $L_2 = \{ g^i(h(g^i(a))) \mid i \geq 0 \}$.

Proposition

$L_2 \in \mathscr{L}(\text{spRT}) \setminus \mathscr{L}(\text{gr-RWWT})$.

Let $L_3 = \{ f(g^i(a), g^i(a)), f(g^i(a), g^{2i}(b)) \mid i \geq 0 \}$.

Proposition

$L_3 \in \mathscr{L}(\text{gr-RT}) \setminus \mathscr{L}(\text{spRWT})$.

Single-Path Versus Ground Rewrite Restriction

Let $L_2 = \{ g^i(h(g^i(a))) \mid i \geq 0 \}$.

Proposition

$L_2 \in \mathscr{L}(\text{spRT}) \setminus \mathscr{L}(\text{gr-RWWT})$.

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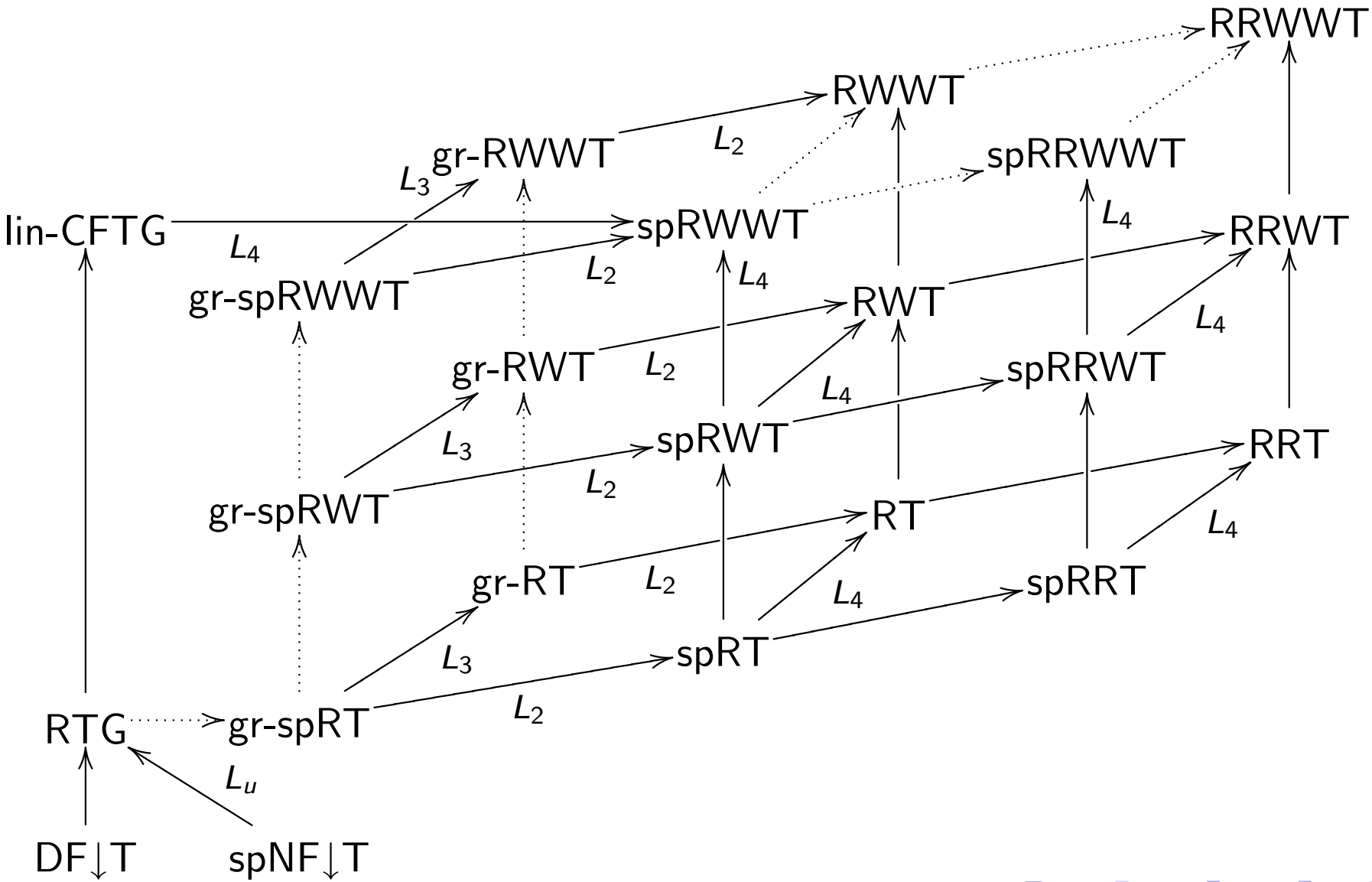
Proposition

$L_3 \in \mathscr{L}(\text{gr-RT}) \setminus \mathscr{L}(\text{spRWT})$.

Taxonomy of Restarting Tree Automata



Conclusion

Open Questions:

- Is any of the following inclusions proper:

$$\mathscr{L}(\text{gr-RT}) \subseteq \mathscr{L}(\text{gr-RWT}) \subseteq \mathscr{L}(\text{gr-RWWT})?$$

- Does the equality $\mathscr{L}(\text{RTG}) = \mathscr{L}(\text{gr-spRWT})$ hold?
  If so, $\mathscr{L}(\text{RTG}) = \mathscr{L}(\text{gr-spRWWT})$ follows easily.
- For each (gr-)spRWWT-automaton $\mathcal{A}$, is there an equivalent (gr-)spRWWT-automaton $\mathcal{B}$ such that $L(\mathcal{B}_{S_G})$ is finite?

Thank you! Questions?

Conclusion

Open Questions:

- Is any of the following inclusions proper:

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- Does the equality $\mathscr{L}(\text{RTG}) = \mathscr{L}(\text{gr-spRWT})$ hold?
  If so, $\mathscr{L}(\text{RTG}) = \mathscr{L}(\text{gr-spRWWT})$ follows easily.
- For each (gr-)spRWWT-automaton $\mathcal{A}$, is there an equivalent (gr-)spRWWT-automaton $\mathcal{B}$ such that $L(\mathcal{B}_{S_G})$ is finite?

Thank you! Questions?

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